Texture Based Classification of Seismic Image Patches Using Topological Data Analysis

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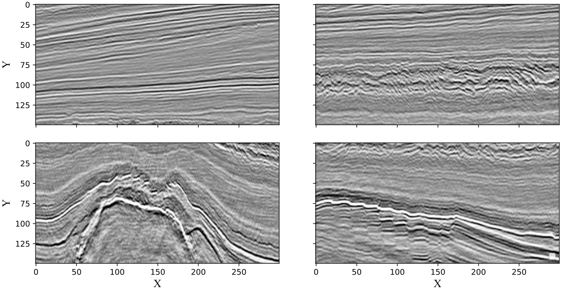
Seismic Texture Classification

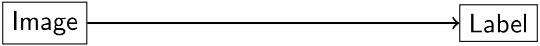


→ Flat Horizon
Chaotic Horizon
Fault
Salt Body

Example image patches from the LANDMASS-2 data set [4].

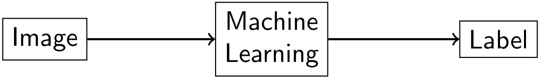
This talk: one way to do this using topological data analysis.

Seismic Texture Classification



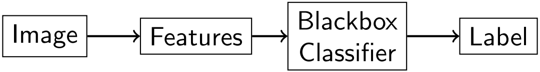
What we want

Seismic Texture Classification



A popular approach.

Seismic Texture Classification



Now we have a road map

Texture

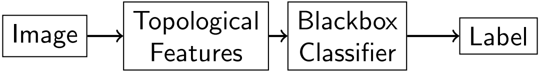
Characteristics of texture

- Area with similar 'look' - no *one* thing to focus on
- Repetitive/recurrent (but not necessarily periodic)
- Scale, translation, rotation, deformation invariant

Machine learning with images

- Treating each pixel as a feature is typically not well-suited for classification tasks.
- Most existing methods use derived features (either hand-crafted or learned).
- We'll show how topological features capture the properties of texture.

Seismic Texture Classification



Where we're headed

Topology

Topology:

- Scale, translation, rotation, deformation invariant
- Good at capturing qualitative information

Topological Data Analysis:

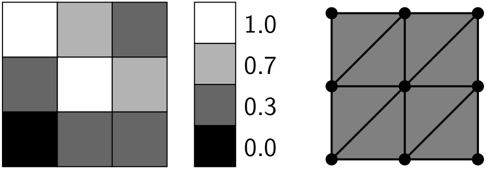
- Generally: tools to use and understand topological structure in data
- Tools to turn topology into features (real numbers)

Topology in Images

To use topology on images, we need a way to turn an image into a topological space. We represent the space on a computer using points, edges, and triangles (simplicial complex).

One way to do this:

- Pixels become points in the space
- Adjacent pixels are connected by an edge
- Diagonal edges added by Freudenthal triangulation
- 3 adjacent pixels are spanned by a triangle

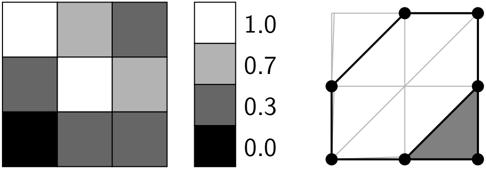


Topology in Images

Topological spaces created from all pixels in the image have the same topology as the ‘canvas’ (e.g. rectangle). This won’t help with classifying different images.

A more interesting topological space:

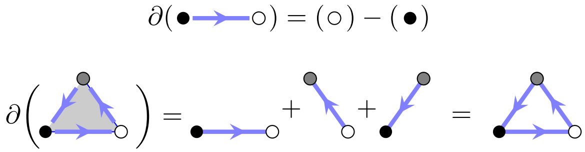
- Choose some pixel value t
- Only points with pixel values are $\leq t$ are used
- Only edges with both endpoints are included
- Only triangles with boundary edges are included



$t = 0.7$

Homology

Consider formal linear combinations of vertices/edges/triangles in a space X. This produces a set of vector spaces $C_k(X)$ ($k = 0$ for vertices, $k = 1$ for edges...). There are linear boundary maps $\partial_k : C_k(X) \to C_{k-1}(X)$



with the property $\partial \circ \partial = 0$.
Homology of the space in dimension k is the quotient

$$H_k(X) = \ker(\partial_k) / \operatorname{img}(\partial_{k+1})$$

$\dim H_0$ counts clusters that are not connected. $\dim H_1$ counts cycles (holes) that are not boundaries.

Filtrations

A filtration is

- a set of spaces X_t, $t \in I \subseteq \mathbb{R}$
- the spaces satisfy $X_s \subseteq X_t$ if $s \leq t$

We obtain a filtration from an image by constructing spaces X_t for every pixel value t in the image.

X_t is the space constructed from pixel values $\leq t$.

Persistent Homology

Persistent homology is a tool from topological data analysis

Input: A filtration X_t

Output: A collection of pairs of real numbers for each dimension k in the space

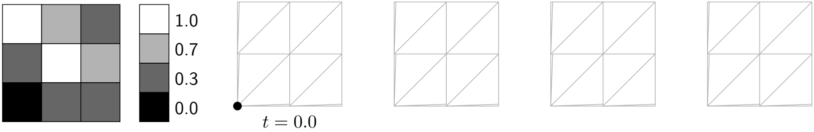
$$PH_k(X_t) = \{(b_j, d_j)\}$$

These are called birth-death pairs, and track how homology changes over the filtration.

Properties:

- homotopy invariant (deformation, rotation, translation)
- stable to perturbations of pixel values

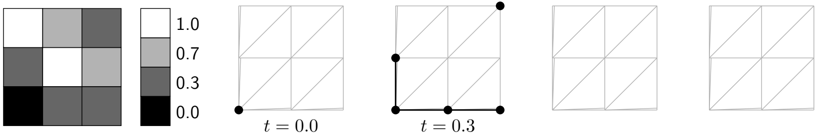
Example



Left: Example image Right: Corresponding filtration

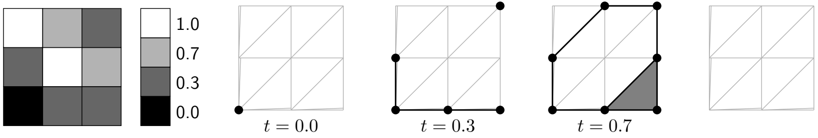
At $t = 0$, a single point appears...

Example



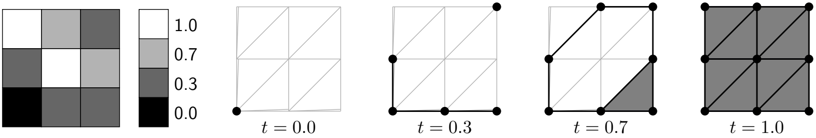
At $t = 0.3$, several points connect to the first point, and a new component emerges

Example



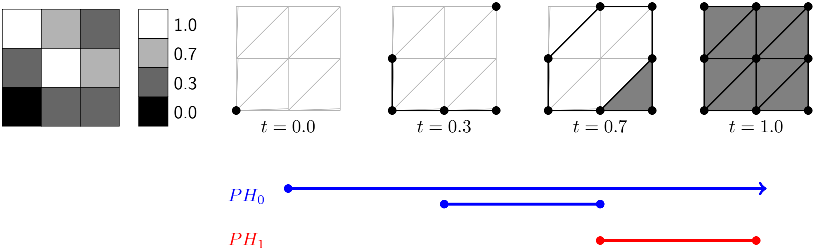
At $t = 0.7$, the two components join, and a hole appears.
We also see our first triangle.

Example



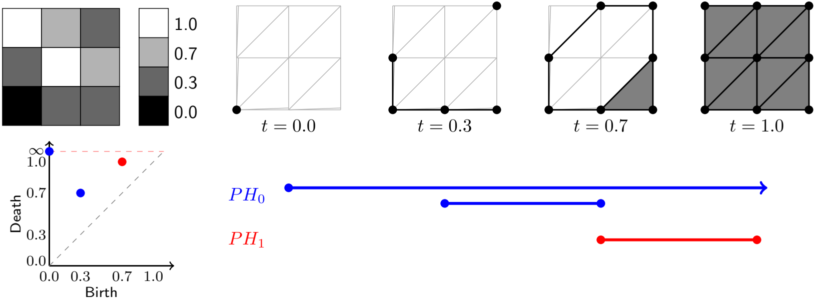
At $t = 1.0$, all points are now present, and all edges and triangles fill in the space

Example



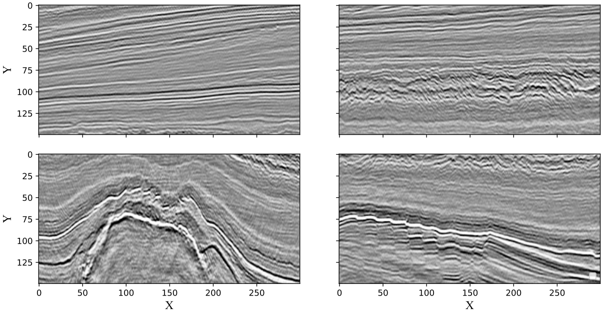
Persistence Barcode:
Information about how components appear and merge is encoded in PH_0
Information about how holes appear and fill in is encoded in PH_1

Example

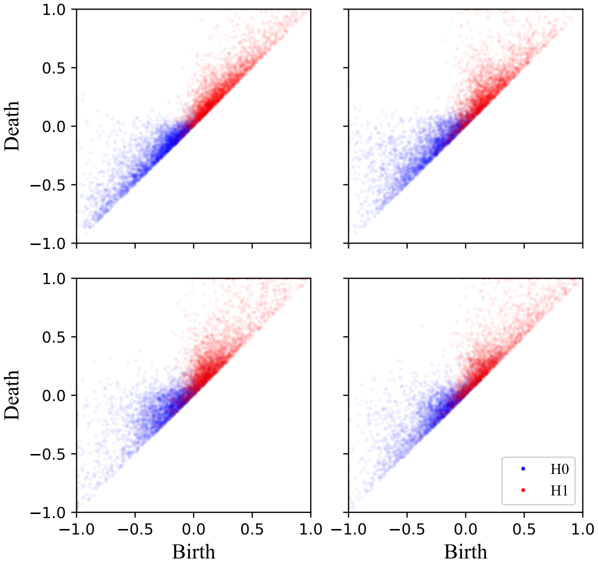


Persistence Diagram:
The start and endpoints of the barcode are plotted in the plane.
Each point is referred to as a birth-death pair.

Seismic Texture Classification



Seismic Texture Classification



Features

We went from an image to a set of 2D points

- Every image produces a different number of birth-death pairs
- We want a standard number of features

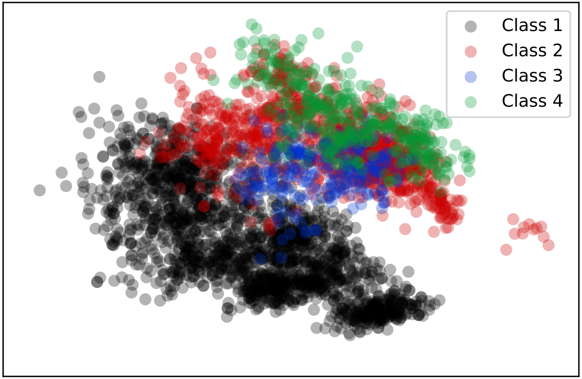
One approach - polynomial functions [2]:

$$p(\alpha; \{(b_i, d_i)\}_{i \in J}) = \frac{1}{|J|} \sum_{i \in J} \sum_{j,k} \alpha_{j,k} (d_i - b_i)^j (d_i + b_i)^k$$

Choose several different α, and we have as many features. We use $\alpha = \delta_{ij}, (i, j) \in \{0, 1, 2, 3\}^2 - (0, 0)$ for homology dimensions 0 and 1. This produces $15 \times 2 = 30$ features.

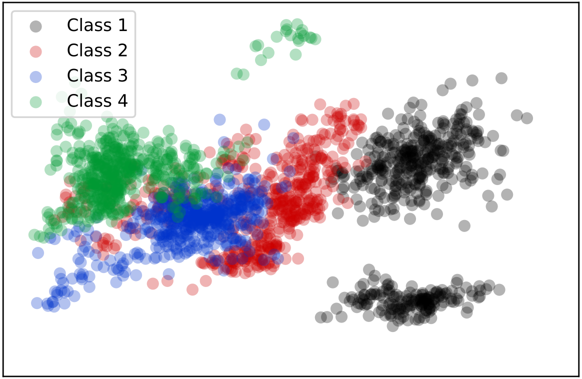
These look a lot like empirical expectations of functions using point measures in $\mathbb{R}^2$.

LANDMASS-1 Features



Projection of topological features into top 2 principal components. Each point is an image in the LANDMASS-1 data set.

LANDMASS-2 Features



Projection of topological features into top 2 principal components. Each point is an image in the LANDMASS-2 data set.

GLCM Features

We also investigate the use of grey level co-occurence matrices (GLCM) features to produce new images.

Each GLCM feature produces a new image, where pixel values depend on a small patch.

GLCM Features: Entropy, Energy, Variance, Correlation, Mean, Max Probability...

We use these additional GLCM images to produce additional persistence diagrams for each input image, and use the same polynomial features.

Method

- Split data into train and test
- Produce persistence diagrams for each image
- Produce polynomial features from each persistence diagram
- Train classifier on polynomial features
- 3 Black box classifiers tested
  - Multi-class SVM
  - Random Forest
  - Neural network

Persistent homology calculations done with GUDHI [5] SVM and RF use scikit-learn [6] NN uses tensorflow [7].

Results

Attribute	SVM Accuracy
Raw Image	99.8 / 75.2 / 0.0 / 0.0 100.0 / 55.0 / 88.3 / 74.3
GLCM Mean	100.0 / 18.6 / 34.1 / 29.3 62.7 / 19.0 / 4.0 / 100.0
RMS Amplitude	100.0 / 1.0 / 0.0 / 0.0 74.7 / 85.7 / 71.3 / 61.7
GLCM Correlation	100.0 / 0.0 / 0.0 / 0.0 64.7 / 32.0 / 89.3 / 32.3
GLCM Variance	96.6 / 94.1 / 92.8 / 67.7 97.3 / 93.3 / 91.7 / 87.0

In each row: top = LANDMASS-1, bottom = LANDMASS-2,
color corresponds to class

Results

Attribute	RF Accuracy
Raw	99.9 / 98.6 / 95.2 / 93.3
Image	100.0 / 98.0 / 100.0 / 96.3
GLCM	99.9 / 97.9 / 82.1 / 93.3
Mean	100.0 / 97.0 / 97.3 / 91.7
RMS	99.3 / 96.1 / 88.0 / 82.0
Amplitude	99.7 / 96.0 / 96.0 / 91.7
GLCM	99.3 / 94.9 / 80.8 / 91.2
Correlation	99.7 / 93.7 / 92.0 / 97.0
GLCM	98.5 / 95.7 / 96.3 / 74.0
Variance	99.0 / 95.3 / 96.7 / 89.7

In each row: top = LANDMASS-1, bottom = LANDMASS-2, color corresponds to class

Results

Attribute	NN Accuracy
Raw Image	100.0 / 99.6 / 99.7 / 98.4 100.0 / 100.0 / 99.0 / 95.0
GLCM Mean	100.0 / 97.8 / 92.8 / 97.0 100.0 / 96.0 / 95.7 / 96.3
RMS Amplitude	99.5 / 99.1 / 96.3 / 91.5 99.7 / 99.0 / 93.7 / 91.3
GLCM Correlation	99.8 / 93.6 / 87.7 / 96.7 100.0 / 95.7 / 93.7 / 98.3
GLCM Variance	99.3 / 98.3 / 98.1 / 87.3 99.7 / 99.0 / 99.3 / 95.0

In each row: top = LANDMASS-1, bottom = LANDMASS-2,
color corresponds to class

Conclusions

- Topological features potentially give high accuracy
- SVM had poorest performance. RF and NN did much better, probably due to nonlinear decision boundaries
- some (not all) GLCM features also do well when combined with topological features

Some next steps

- Analysis of space of topological features (not just classification accuracy)
- How many polynomial features do we need?
- (Further out) 3D images

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